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Optimizing Distributed Structures for Maximum Flutter Load

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Introduction

INTEREST in design optimization of distributed structures is increasing.[1] A survey of optimization under nonconservative loading[2] reveals, however, that virtually all work done in the area has been restricted to very simple structural domains and very simplified design spaces. Besides methods from optimal control theory, typical approaches have consisted in assuming a design function of simple form for the whole domain and optimizing its coefficients through a search in few dimensions[3] or in using a finite-element model and optimizing the location of its nodes through discrete sensitivity methods.[4] Such approaches allow only relative optimization to take place within a restricted design space which cannot be enlarged without problems: completeness of the general design function and convergence for the former, and size of the supporting analysis for the latter.

This Note describes a method, based on ideas similar to some in Ref. 5, to find essentially true optimal designs for a wide class of distributed structures undergoing flutter in the presence of damping. Design has the form of a general function described by its values at points, whose number can be increased at will with little effort and without affecting the size of analysis. We solve a hitherto unsolved problem, and indicate how this method may be combined with the finite-element method to solve large-scale problems in complex structural domains.

Analysis

Autonomous vibrations and flutter of many practical distributed structures are governed by a quadratic eigenvalue problem of the form

$$[K+pE+\lambda D+\lambda^2 M]u=0 \tag{1}$$

where K, E, D, and M are real linear differential operators representing stiffness, loading, damping, and mass properties respectively, p is a real loading parameter, and the eigenfrequency $\lambda=\beta+i\omega$ has complex right and left eigenfunctions (eigenmodes) u and v. Operator E is assumed to be nonself-adjoint.

Approximate analysis replaces Eq. (1) by a matrix equation of the same form, whose solution[6] yields *characteristic curves* as those in Fig. 1. Such curves provide a direct and global image of the frequency spectrum and stability status of the structure at all loads. It is well known that the onset of flutter occurs when at least one of these curves hits the plane $Re(\lambda)=0$ at a *flutter point* (p_{fl},ω_{fl}); we must make this point highest for a given total mass by properly modifying a design function h entering the definition of the operators.

Design Sensitivity Analysis

Let the design function h be given an arbitrary variation δh; then

$$\delta(v,[K+pE+\lambda D+\lambda^2 M]u)=0 \tag{2}$$

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where (.,.) is a weak form[7] of operator Eq. (1) and is assumed to be differentiable with respect to h. The solutions of Eq. (1) will vary as in Fig. 1, and the corresponding variation of the flutter point will be such that

$$\Delta_R+i\Delta_I+(E_R+iE_I)\delta p_{fl}+(F_R+iF_I)i\delta\omega_{fl}=0 \tag{3}$$

where subscripts R and I denote real and imaginary parts, and where, formally,

$$\Delta_R+i\Delta_I=(v_{fl},\ [\delta K+p_{fl}\delta E+i\omega_{fl}\delta D-\omega_{fl}^2\delta M]u_{fl}) \tag{4}$$

and

$$E_R+iE_I=(v_{fl},\ Eu_{fl}) \tag{5}$$

$$F_R+iF_I=(v_{fl},\ [D+2i\omega_{fl}M]u_{fl}) \tag{6}$$

Solution of Eq. (3) readily yields the variation of the flutter point due to δh as

$$\delta p_{fl}=-(F_R\Delta_R+F_I\Delta_I)/(E_RF_R+E_IF_I) \tag{7}$$

$$\delta\omega_{fl}=(E_I\Delta_R+E_R\Delta_I)/(E_RF_R+E_IF_I) \tag{8}$$

Distributed design sensitivity analysis was studied by Farshad[8] for buckling and vibration problems, and by the authors[9] for flutter problems with no damping.

Gradient Projection

Consider a functional p (critical load, frequency, etc.) of the general form

$$p=p[h(x)]=\int_\Omega P[x,h(x)]\mathrm{d}x \tag{9}$$

where h is the design function and P is a scalar function. We consider here maximization of p under integral equality constraints of the similar form

$$g_k[h(x)]=\int_\Omega G_k[x,h(x)]\mathrm{d}\Omega=\text{const}\quad(k=1,\ldots,n) \tag{10}$$

When P and all G_k are differentiable with respect to h one has

$$\delta p=\int_\Omega P_{,h}\delta h\mathrm{d}\Omega \tag{11}$$

and

$$\delta g_k=\int_\Omega G_{k,h}\delta h\mathrm{d}\Omega=0 \tag{12}$$

Functions $P_{,h}$ and $G_{k,h}$ are the gradients of functionals p and g_k in design function space. For example, the gradient of a flutter load will be, from Eq. (7),

$$P_{fl,h}=-(F_RH_R+F_IH_I)/(E_RF_R+E_IF_I) \tag{13}$$

where function H is the integrand, excluding δh, of the (integral) expression

$$(v_{fl},\ [K_{,h}+p_{fl}E_{,h}+i\omega_{fl}D_{,h}-\omega_{fl}^2M_{,h}]u_{fl}\delta h) \tag{14}$$

In order to improve p, seek a function δh with

$$\|\delta h\|=\left[\int_\Omega(\delta h)^2\mathrm{d}\Omega\right]^{1/2}=\epsilon>0$$

and maximizing functional Eq. (11) under constraints Eq.

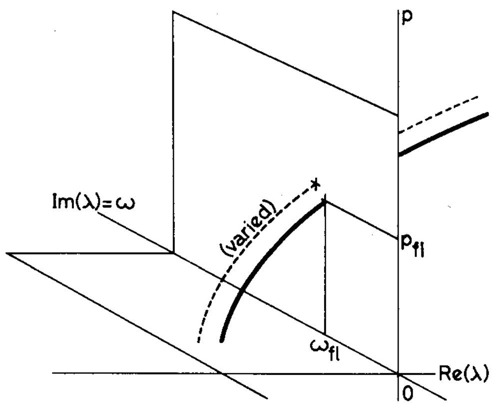

Fig. 1 Typical characteristic curve and flutter point.

(12). Form the functional

$$\int_\Omega \Big(P_{,h}-\sum_k \mu_k G_{k,h}\Big)\delta h \mathrm{d}\Omega \tag{15}$$

where the μ_k are Lagrange multipliers. From the Schwarz inequality, the function

$$\delta h=\alpha\Big(P_{,h}-\sum_k \mu_k G_{k,h}\Big) \tag{16}$$

is the distributed projected gradient sought; α is a real scalar. All μ_k are found by substituting Eq. (16) into Eq. (12) and solving the linear system

$$\sum_k \int_\Omega G_{i,h}G_{k,h}\mathrm{d}\Omega\mu_k=\int_\Omega G_{i,h}P_{,h}\mathrm{d}\Omega \quad (i=1,...,n) \tag{17}$$

To determine the factor α, take norms in Eq. (16), giving

$$\alpha=\epsilon\Big/\Big\|P_{,h}-\sum_k \mu_k G_{k,h}\Big\| \tag{18}$$

or aim at a given increase Δp by substituting Eq. (16) into Eq. (11), giving

$$\alpha=\Delta p\Big/\int_\Omega P_{,h}\Big(P_{,h}-\sum_k \mu_k G_{k,h}\Big)\mathrm{d}\Omega \tag{19}$$

To correct constraint violations Δg_k, a minimum-norm variation $\delta' h$ such that

$$\int_\Omega G_{k,h}\delta' h\mathrm{d}\Omega=-\Delta g_k \quad (k=1,...,n) \tag{20}$$

is necessary. The Euler equation of an appropriate augmented functional yields

$$\delta' h=\sum_k \nu_k G_{k,h} \tag{21}$$

Substituting Eq. (21) into Eq. (20), all ν_k are found from the linear system

$$\sum_k \int_\Omega G_{i,h}G_{k,h}\mathrm{d}\Omega\nu_k=-\Delta g_i \quad (i=1,...,n) \tag{22}$$

Example 1. Consider maximizing functional Eq. (9) when $\Omega=[0,\ell]$ and under the constraint

$$g=\int_0^\ell h(x)\,\mathrm{d}x=\text{const}$$

Then

$$\delta g=\int_0^\ell \delta h \mathrm{d}x=0$$

so that $G_{,h}=1$ identically, yielding

$$\mu=\int_0^\ell P_{,h}\mathrm{d}x/\ell$$

and

$$\alpha=\Delta p\Big/\Big[\int_0^\ell P_{,h}^2\mathrm{d}x-\Big(\int_0^\ell P_{,h}\mathrm{d}x\Big)^2\Big/\ell\Big]$$

from which

$$\delta h(x)=\Delta p\Big(P_{,h}-\int_0^\ell P_{,h}\mathrm{d}x/\ell\Big)\Big/\Big[\int_0^\ell P_{,h}^2\mathrm{d}x-\Big(\int_0^\ell P_{,h}\mathrm{d}x\Big)^2\Big/\ell\Big] \tag{23}$$

Any constraint violation can be corrected using $\delta' h=-\Delta g/\ell$.

Example 2. In addition to the preceding constraint, iterations are now to be performed so that $\Delta q=\theta\Delta p$, where q is a functional $\int_0^\ell Q\mathrm{d}x$ and θ is a scalar. Then,

$$\delta h(x)=\alpha[(1+\mu_1\theta)P_{,h}-\mu_1 Q_{,h}-\mu_2] \tag{24}$$

$$\alpha=\Delta p\Big/\int_0^\ell P_{,h}[(1+\mu_1\theta)P_{,h}-\mu_1 Q_{,h}-\mu_2]\mathrm{d}x \tag{25}$$

$$\mu_1=\Big(\ell\int_0^\ell R_{,h}P_{,h}\mathrm{d}x-\int_0^\ell R_{,h}\mathrm{d}x\int_0^\ell P_{,h}\mathrm{d}x\Big)\Big/D \tag{26}$$

$$\mu_2=\Big(\int_0^\ell R_{,h}^2\mathrm{d}x\int_0^\ell P_{,h}\mathrm{d}x-\int_0^\ell R_{,h}\mathrm{d}x\int_0^\ell R_{,h}P_{,h}\mathrm{d}x\Big)\Big/D \tag{27}$$

$$\delta' h=\Big[\ell(\Delta q-\delta q)+\int_0^\ell R_{,h}\mathrm{d}x\Delta g\Big]Q_{,h}\Big/D$$
$$-\Big[\int_0^\ell R_{,h}\mathrm{d}x(\Delta q-\delta q)+\int_0^\ell R_{,h}^2\mathrm{d}x\Delta g\Big]\Big/D \tag{28}$$

with

$$D=\ell\int_0^\ell R_{,h}^2\mathrm{d}x-\Big(\int_0^\ell R_{,h}\mathrm{d}x\Big)^2 \tag{29}$$

$$R_{,h}=Q_{,h}-\theta P_{,h} \tag{30}$$

where $\delta' h$ corrects the change of q from δq (obtained) to Δq (desired).

Example

Beck's column with Kelvin-Voigt internal damping η has the familiar equation

$$(EIu'')''+pu''+\lambda\eta(EIu'')''+\lambda^2 mu=0 \tag{31}$$

Let overbars denote complex conjugates; a weak form of Eq. (31) is in this case

$$\int_0^\ell (\bar{v}''EIu''+\bar{v}pu''+\lambda\eta\bar{v}''EIu''+\lambda^2\bar{v}mu)\,\mathrm{d}x=0 \tag{32}$$

whose matrix solution for a uniform column of unit length, using eight polynomial coordinate functions and Simpson integration with 101 points, is shown in Fig. 2.

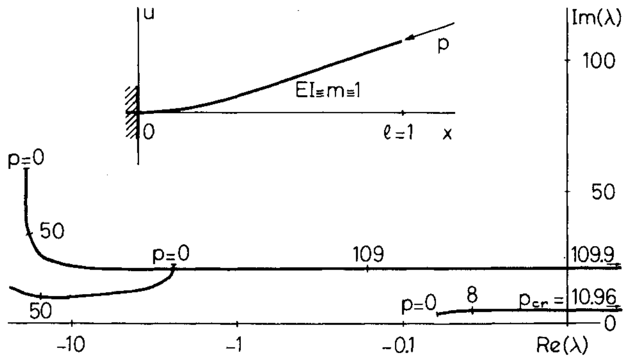

Fig. 2 Uniform Beck's column with internal damping ($\eta = 0.01$) and its characteristic curves.

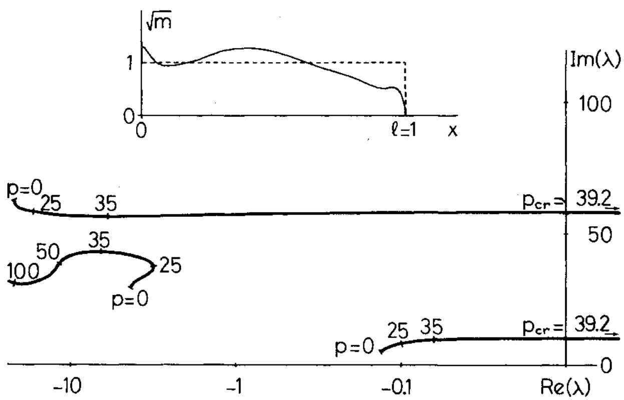

Fig. 3 Optimal Beck's column with internal damping ($\eta = 0.01$) and its characteristic curves (in 18 iterations).

Letting $EI = m^2$, maximization of p was performed by Eqs. (13) and (23-30) in order to control or prevent the drop of the second flutter load at each iteration. In the final result shown in Fig. 3, the optimal critical load is a double flutter load. The increase in critical load is more than 250%, against about 100% in Ref. 3.

Concluding Remarks

Due to lack of space we remark only the following. 1) This method of flutter optimization, including sensitivity formulas [Eqs. (7) and (8)] and control of the second flutter load, is believed new. 2) A finely discretized design function (not parameters) was optimized by explicit distributed gradient projection. 3) The size of the analysis is not affected by the number of design data points, which should be large. 4) One can readily add inequality-type or pointwise constraints. 5) Large-scale application requires the development of (possibly low-order) two- and three-dimensional finite elements with outer boundaries defined by relatively many design points; interpolation on these boundaries can be quite rudimentary.

Acknowledgment

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